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Duality of type-II 7-branes and 8-branes ★

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Abstract

We present a version of ten-dimensional IIA supergravity containing a 9-form potential for which the field equations are equivalent to those of the standard, massless, IIA theory for vanishing 10-form field strength, F_{10}, and to those of the 'massive' IIA theory for non-vanishing F_{10}. We present a multi 8-brane solution of these equations that generalizes the 8-brane of Polchinski and Witten. We show that this solution is T-dual to a new multi 7-brane solution of S^1 compactified IIB supergravity, and that the latter is T-dual to the IIA 6-brane. When combined with the $Sl(2;\mathbb{Z})$ U-duality of the type-IIB superstring, the T-duality between type-II 7-branes and 8-branes implies a quantization of the cosmological constant of type-IIA superstring theory. These results are made possible by the construction of a new *massive* $N = 2$ $D = 9$ supergravity theory. We also discuss the 11-dimensional interpretation of these type-II p-branes.

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1. Introduction

Recent advances in our understanding of non-perturbative superstring theory have led to the establishment of many connections between hitherto unrelated superstring theories. Many of these connections involve p-brane solutions of the respective supergravity theories that couple to the $(p + 1)$-form potentials in the Ramond-Ramond (RR) sector. Most of these RR p-branes, and all of the IIA ones, are singular as solutions of

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ten-dimensional ($D = 10$) supergravity, so their status in superstring theory was unclear until recently (see Ref. [1] for a recent review). It now appears [2] that the RR p-branes of type-II supergravity theories have their place in type-II superstring theory as 'Dirichlet-branes', or 'D-branes' [3]. These include the p-branes for $p = 0,2,4,6$ in the type-IIA case and the p-branes for $p = 1,3,5$ in the type-IIB case. However, they also include a type-IIB 7-brane and a type-IIA 8-brane and it is possible to view the $D = 10$ space-time as a type-IIB 9-brane [2]. Note that since the dual of a p-brane in $D = 10$ is a $(6-p)$-brane, only p-branes with $p \leq 6$ have duals with $p \geq 0$ for which a standard (Minkowski space) interpretation is available [1], so the p-branes with $p \geq 7$ have implications that are qualitatively different from those with $p \leq 6$. This difference is also apparent in the p-brane solutions of the effective IIA or IIB supergravity field equations. These solutions generally involve a function that is harmonic on the $(9-p)$-space 'transverse' to the $(p+1)$-dimensional worldvolume of the p-brane. For $p \leq 6$ the transverse space has dimension 3 or greater so there exist harmonic functions that are constant at infinity, but for $p \geq 7$ the transverse space has dimension 2 or less and the asymptotic properties are therefore qualitatively different. Partly for this reason little attention has been given so far to the $(p \geq 7)$-branes.

Since p-branes couple naturally to $(p+1)$-form potentials, one expects to find a stable p-brane solution of a supergravity theory only if it includes a $(p+1)$-form potential. From this perspective the IIB $D = 10$ 7-brane is the most straightforward of the $p \geq 7$ cases because the pseudo-scalar field of IIB supergravity can be exchanged for its 8-form dual. Indeed, a type-IIB 7-brane solution has recently been described [4]; it can be viewed as a dimensional 'oxidation' of the 'stringy cosmic string' solution of Ref. [5]. However, this class of 7-brane solutions is specific to the *uncompactified* IIB supergravity and is therefore not expected to be related by T-duality to other type-II p-branes. Here we shall present a new class of multi 7-brane solutions of the S^1-compactified IIB supergravity. In the decompactification limit the new solutions reduce to the trivial $D = 10$ Minkowski space-time solution. Nevertheless, as we shall see, these solutions are T-dual to both the 6-brane and the IIA 8-brane solutions of the IIA theory.

The existence of an 8-brane solution of IIA supergravity is obscured by the absence of a 9-form potential, A_9, in the standard IIA supergravity theory. However, there is one in type-IIA superstring theory [2] and this suggests that it should be possible to introduce one into the IIA supergravity theory. The 9-form potential would have a 10-form field strength F_{10}. Assuming a standard kinetic term of the form F_{10}^2, the inclusion of this field does not lead to any additional degrees of freedom (per space-time point) and so is not immediately ruled out by supersymmetry considerations, but it allows the introduction of a cosmological constant, as explained many years ago in the context of a 4-form field strength in four-dimensional field theories [6,7]. As it happens, a version of type-IIA supergravity theory with a cosmological constant was constructed (up to quartic fermion terms) some time ago by Romans [8], who called it the 'massive' IIA supergravity theory; the complete construction via superspace methods was found

[1] The IIB 7-brane has a (-1)-brane dual, but the latter has an interpretation as an instanton [4].

subsequently [9]. It has been argued that the existence of the massive IIA supergravity is related to the existence of the 9-form potential of type-IIA superstring theory [2]. Here we shall confirm this suggestion by reformulating the massive IIA supergravity through the introduction of a 9-form potential[2]. The new theory has the advantage that its solutions include those of both the massless and the massive IIA theory. We propose this new IIA supergravity theory as the effective field theory of the type-IIA superstring, allowing for the 9-form potential. It has been suggested [2,10] that the expectation value of the dual of this 10-form field strength should be interpreted as the cosmological constant of the massive IIA supergravity theory. One result of this paper is the determination of the precise relation between these quantities; they are conjugate variables in a sense discussed previously in the $D = 4$ context [11].

The massive IIA supergravity theory has the peculiarity that $D = 10$ Minkowski space-time is *not* a solution of the field equations (and neither is the product of $D = 4$ Minkowski space-time with a Calabi-Yau space). Various Kaluza-Klein (KK) type solutions were found by Romans, but none of them were supersymmetric, i.e. his solutions break all the supersymmetries. A supersymmetric multi 8-brane configuration was recently proposed as a solution of the Killing spinor condition in an appropriate bosonic background [12]. We verify that this is a solution of the field equations of the new IIA supergravity theory and we present a generalization of it. The solutions are all singular at the 'centers' of the metric, i.e. the 8-brane positions, but this is a general feature of RR p-branes.

It is known that after compactification on S^1 the *perturbative* type-IIA and type-IIB superstrings are equivalent [3,13], being related by a $\mathbb{Z}_2$ T-duality transformation that takes the radius R of the S^1 of one superstring theory into a radius $1/R$, in appropriate units, of the other superstring theory. It follows that the *same* effective $N = 2$ $D = 9$ field theory should be obtained by dimensional reduction of either the IIA or IIB theory in $D = 10$, and this is in fact the case [14]. If this $\mathbb{Z}_2$ T-duality is valid non-perturbatively too, then p-brane solutions of the IIA theory must correspond to p-brane solutions of the IIB theory and vice versa, in the sense that there are solutions of either the IIA or the IIB theory that reduce to the same solution of the S^1-compactified theory. In particular the double dimensional reduction to $D = 9$ of a given IIA 8-brane should be equivalent to the direct reduction to $D = 9$ of some IIB 7-brane. There is a potential difficulty in verifying this because the relevant $D = 9$ theory must be a *massive* $N = 2$ supergravity theory. It is not too difficult to see how to obtain a massive $N = 2$ $D = 9$ supergravity from the massive $D = 10$ IIA theory but it is not so obvious how the resulting theory may also be obtained from the (necessarily massless) $D = 10$ IIB theory, although it must be possible if T-duality is to be valid non-perturbatively. As we shall show, it is possible by an application of a mechanism for obtaining a massive theory in a lower dimension from a massless one in a higher dimension. This mechanism is essentially that of Scherk and Schwarz [15] but in our case supersymmetry is preserved by the

[2] Strictly speaking we do this only for the bosonic lagrangian, but the method guarantees the existence of a fermionic extension to a full supergravity theory.

reduction. This result allows us to map 8-brane solutions of the $D = 10$ IIA theory into 7-brane solutions of the IIB theory, and vice versa.

These IIB 7-brane and IIA 8-brane solutions may be seen as the effective field theory realization of the associated D-branes of the corresponding type-II superstring theory. In this context, the $Sl(2;\mathbb{R})$ symmetry of the IIB supergravity is expected to be replaced by an $Sl(2;\mathbb{Z})$ U-duality [16], which amounts to an identification of points in the space $Sl(2,\mathbb{R})/U(1)$ of IIB vacua that differ by the action of $Sl(2,\mathbb{Z})$. One interesting consequence of this IIB duality, when combined with the T-duality of the 7-brane and 8-brane, is a quantization of the cosmological constant of the S^1-compactified IIA superstring theory [3].

The organization of this article is as follows. In Section 2, we begin with a review of the massive IIA supergravity, introducing some simplifications. In Section 3, we construct the new formulation of the bosonic sector of this theory, incorporating the 9-form gauge field A_9, in which the cosmological constant emerges as an integration constant. In Section 4, we construct supersymmetric multi 8-brane solutions of the massive IIA supergravity theory, some of which are asymptotically flat. In Section 5, we show how both the massive IIA supergravity and the (massless) IIB supergravity theories may be dimensionally reduced to yield a new $D = 9$ $N = 2$ massive supergravity theory. We then use this to establish the massive type-II T-duality rules. In Section 6, we construct the most general 7-brane solutions of the IIB theory that are both compatible with the KK ansatz and preserve half the supersymmetry. We then show that the massless T-duality transformations take this solution to the IIA 6-brane while the massive T-duality transformations take it to the IIA 8-brane solution. In Section 7 we further comment on the relation to type-IIA superstring theory and the quantization of the cosmological constant, and on the connection to $D = 11$ 'M-theory'. Finally, in Appendix A we give a simplified formulation of the supersymmetry transformations of IIB supergravity.

2. The massive $D = 10$ IIA supergravity

The bosonic field content of the massive IIA $D = 10$ supergravity theory comprises (in our notation) the (Einstein) metric $g^{(E)}$, the dilaton σ, a massive 2-form tensor field B' and a 3-form potential C'. One introduces the field strengths

$$G = 4dC' + 6m(B')^2, \qquad H = 3dB', \tag{2.1}$$

where m is a mass parameter. The lagrangian for these fields is [8]

$$\begin{aligned}\mathcal{L} = \sqrt{-g^{(E)}}\,\big[R_{(E)} &- \tfrac{1}{2}|\partial\sigma|^2 - \tfrac{1}{3}e^{-\sigma}|H|^2 - \tfrac{1}{12}e^{\sigma/2}|G|^2 - m^2 e^{3\sigma/2}|B'|^2 \\ &- \tfrac{1}{2}m^2 e^{5\sigma/2}\big] + \tfrac{1}{9}\varepsilon\left[dC'dC'B' + mdC'(B')^3 + \tfrac{9}{20}m^2(B')^5\right].\end{aligned} \tag{2.2}$$

The notation for forms being used here is that a q-form Q has components $Q_{M_1,\dots,M_q}$ given by

[3] We thank John Schwarz for suggesting this possibility to us.

$$Q = Q_{M_1,\dots,M_q} dx^{M_1} \wedge \dots \wedge dx^{M_q}. \tag{2.3}$$

Thus, the $\frac{1}{9}\varepsilon dC' dC' B'$ term in (2.2) is shorthand for

$$\tfrac{1}{9}\varepsilon^{M_1,\dots,M_{10}} \partial_{M_1} C'_{M_2M_3M_4} \partial_{M_5} C'_{M_6M_7M_8} B_{M_9M_{10}}. \tag{2.4}$$

As explained in Ref. [8] the massless limit is not found by simply setting $m = 0$ in (2.2) because the supersymmetry transformations involve terms containing m^{-1}. Instead, one first makes the field redefinitions

$$B' = B + \frac{2}{m} dA, \qquad C' = \tilde{C} - \frac{6}{m} A dA. \tag{2.5}$$

This redefinition introduces the gauge invariance

$$\delta A = -m\Lambda, \qquad \delta B = 2d\Lambda, \qquad \delta\tilde{C} = 12 A d\Lambda, \tag{2.6}$$

for which the gauge-invariant field strengths are

$$F = 2dA + mB, \qquad H = 3dB, \qquad G = 4d\tilde{C} + 24 B dA + 6mB^2. \tag{2.7}$$

The bosonic lagrangian of the massive IIA theory is now

$$\begin{aligned}\mathcal{L} = \sqrt{-g^{(E)}}\,&\big[R_{(E)} - \tfrac{1}{2}|\partial\sigma|^2 - \tfrac{1}{3}e^{-\sigma}|H|^2 - \tfrac{1}{12}e^{\sigma/2}|G|^2 - e^{3\sigma/2}|F|^2 \\ &- \tfrac{1}{2}m^2 e^{5\sigma/2}\big] + \tfrac{1}{9}\varepsilon\big[d\tilde{C}d\tilde{C}B + 6d\tilde{C}B^2 dA + 12(dA)^2B^3 + md\tilde{C}B^3 + \tfrac{9}{2}mB^4 dA \\ &+ \tfrac{9}{20}m^2(B)^5\big],\end{aligned} \tag{2.8}$$

and the bosonic lagrangian of the massless IIA theory can now be found by taking the $m \to 0$ limit.

The lagrangian (2.8) can be simplified by the further redefinition

$$\tilde{C} = C - 6AB. \tag{2.9}$$

The Λ-gauge transformation of the new 3-form C is

$$\delta C = -6m\Lambda B \tag{2.10}$$

and the gauge-invariant field strengths, F, H, and G are now given by

$$F = 2dA + mB, \qquad H = 3dB, \qquad G = 4dC + 24AdB + 6mB^2. \tag{2.11}$$

At the same time, to make contact with string theory, it is convenient to introduce the string metric

$$g_{MN} = e^{-\sigma/2} g^{(E)}_{MN}. \tag{2.12}$$

The bosonic lagrangian now takes the simple form

$$\begin{aligned}\mathcal{L} = \sqrt{-g}\{&e^{-2\sigma}\big[R + 4|\partial\sigma|^2 - \tfrac{1}{3}|H|^2\big] - |F|^2 - \tfrac{1}{12}|G|^2 - \tfrac{1}{2}m^2\} \\ &+ \tfrac{1}{9}\varepsilon\big[dCdCB + mdCB^3 + \tfrac{9}{20}m^2(B)^5\big]\ .\end{aligned} \tag{2.13}$$

Observe that the final topological term is simply a type of Chern-Simons (CS) term associated with the 11-form G^2H. Thus, the bosonic action of the massive type-IIA supergravity theory can be written as

$$I = \int_{\mathcal{M}_{10}} d^{10}x\,\sqrt{-g}\,\{e^{-2\sigma}\left[R + 4|\partial\sigma|^2 - \tfrac{1}{3}|H|^2\right] - |F|^2 - \tfrac{1}{12}|G|^2 - \tfrac{1}{2}m^2\}$$
$$+\tfrac{1}{9}\int_{\mathcal{M}_{11}} G^2 H, \tag{2.14}$$

where $\mathcal{M}_{11}$ is an 11-manifold with boundary $\mathcal{M}_{10}$. Apart from the cosmological constant, the m-dependent terms in the action can be simply understood as arising from the replacement of the usual m-independent field strengths of the massless type-IIA theory by their m-dependent generalizations (2.11). Furthermore, the m-dependence of these field strengths is completely fixed by the 'Stueckelberg' gauge transformation $\delta A = -m\Lambda$ of A, as are the remaining Λ-transformations. The relation of the constant m appearing in this transformation with the cosmological constant cannot be understood purely within the context of the bosonic lagrangian, but is, of course, fixed by supersymmetry.

Observe that the cosmological constant term in (2.14) is now (in the string metric) independent of the dilaton. This is typical of the RR sector and is consistent with the idea that m can be interpreted as the expectation value of the dual of an RR 10-form field strength. This interpretation would have the additional virtue of restoring the invariance under the discrete symmetry in which all RR fields change sign, a symmetry that is broken by the terms linear in m in (2.13). We shall now show how to reformulate the massive IIA theory along these lines. As we shall see, the cosmological constant is simply related to, but not equal to, the expectation value of the 10-form field strength.

3. $D = 10$ IIA supergravity with 9-form potential

We shall start with the bosonic lagrangian of (2.13). Expanding in powers of m, the associated action $I(m)$ is

$$I(m) = I(0) + \int d^{10}x\left\{2m\sqrt{-g}\left[(dC + 6AdB)\cdot B^2 - 2dA\cdot B\right] + \tfrac{1}{9}m\varepsilon dCB^3\right.$$
$$\left. -\tfrac{1}{2}m^2\sqrt{-g}\left[1 + 2|B|^2 + 6|B^2|^2\right] + \tfrac{1}{20}m^2\varepsilon B^5\right\}, \tag{3.1}$$

where $I(0)$ is the bosonic action of the massless IIA supergravity theory. We now promote the constant m to a field $M(x)$, at the same time introducing a 9-form potential A_9 as a Lagrange multiplier for the constraint $dM = 0$. Omitting a surface term, the Lagrange multiplier term can be rewritten as

$$10\varepsilon dA_9 M. \tag{3.2}$$

The A_9 field equation implies that $M = m$, for some constant m, so the remaining equations are equivalent to those of the massive IIA theory except that the constant m is now arbitrary and that we now have an additional field equation from varying M. This additional equation is

$$\frac{\delta I(M)}{\delta M(x)} = -\varepsilon F_{10}, \tag{3.3}$$

where $I(M)$ is the action (3.1) but with M replacing m, and $F_{10} = 10dA_9$ is the 10-form field strength of A_9. Thus the M equation simply determines the new field strength F_{10}. Observe that the expectation value of (εF_{10}) is *not* equal to the expectation value of $\sqrt{-g}M$, as a matter of principle (although it may equal it in special backgrounds), but is rather the value of the variable canonically conjugate to it.

Note that the gauge and supersymmetry transformations of the action $I(M)$ no longer vanish. However, the variations of $I(M)$ are proportional to dM and can therefore be cancelled by a variation of the new 9-form gauge potential A_9. This determines the gauge and supersymmetry transformations of A_9. The supersymmetry variation will not be needed for our purposes, so we omit it. The Λ-gauge transformation of A_9 found in this way is

$$\delta(\varepsilon A_9) = \tfrac{2}{5}\sqrt{-g}\,[\Lambda\cdot F + (\Lambda B)\cdot G] - \tfrac{1}{30}\varepsilon\big(2\Lambda dCB^2 + M\Lambda B^4\big). \tag{3.4}$$

We now have a new gauge-invariant bosonic action

$$I(M) + \int d^{10}x\, M\varepsilon F_{10}. \tag{3.5}$$

The field M can now be treated as an auxiliary field that can be eliminated via its field equation

$$\sqrt{-g}M = K^{-1}(B)\{\varepsilon(F_{10} + \tfrac{1}{9}dCB^3) + 2\sqrt{-g}[(dC + 6AdB)\cdot B^2 - 2dA\cdot B]\}, \tag{3.6}$$

where

$$K(B) = 1 + 2|B^2| + 6|B^2|^2 - \frac{1}{10\sqrt{-g}}\varepsilon B^5. \tag{3.7}$$

Using this relation in (3.5) we arrive at the lagrangian

$$\begin{aligned}\mathcal{L}_{new} = \mathcal{L}_0 &+ \left[\sqrt{-g}K(B)\right]^{-1}\\ &\times\{\varepsilon(F_{10} + \tfrac{1}{9}dCB^3) + 2\sqrt{-g}[(dC + 6AdB)\cdot B^2 - 2dA\cdot B]\}^2,\end{aligned} \tag{3.8}$$

where $\mathcal{L}_0$ is the bosonic lagrangian of the massless IIA theory. Note the non-polynomial structure of the new lagrangian in the gauge field B. This greatly obscures the Λ-gauge invariance, which is ensured by the very complicated Λ-gauge transformation of A_9.

The full IIA supergravity lagrangian in the new formulation of course requires the inclusion of the fermion terms. Although we have not worked these out, it should be clear from the above construction of the purely bosonic sector that they can be deduced directly from those in the 'old' formulation by following the above steps. Thus, the existence of the full IIA supergravity theory with 9-form potential is guaranteed. Presumably, there also exists an on-shell superspace formulation of the field equations of this new theory, which it would be of interest to find. We leave this problem to future investigations.

4. The IIA $D = 10$ 8-brane

The appearance of the 9-form potential in the above reformulation of the massive IIA supergravity theory suggests the existence of an associated 8-brane solution. We will find solutions of the equations of motion of (3.8) of the form

$$ds^2 = f^2(y)\, dx^\mu dx^\nu \eta_{\mu\nu} + g^2(y)\, dy^2, \qquad \sigma = \sigma(y), \qquad A_9 = A_9(y), \tag{4.1}$$

with all other fields vanishing, and where η is the Minkowski 9-metric. Such a solution will have 9-dimensional Poincaré invariance and hence an interpretation as an 8-brane. We shall further require of such a solution that it preserves some supersymmetry, so we shall begin by considering the variation of the gravitino 1-form ψ and the dilatino λ in the presence of configurations of the above form. The full variations of the massive IIA theory can be found in Ref. [8] in the Einstein frame. They depend on the constant m. In the new theory, this constant is replaced by the function M given in (3.6). Here, however, we shall need the fermion variations in the *string frame*. For $M = 0$ these are implicit in the superspace results of Ref. [17]. For the backgrounds considered here, for which all fermions vanish and $\sqrt{-g}M = \varepsilon F_{10}$, the $M \neq 0$ string-frame fermion variations are most easily deduced from the Einstein-frame results of Ref. [8]. The result is

$$\delta_\epsilon \psi = D\epsilon + \tfrac{1}{8} M e^{\sigma} \Gamma \epsilon, \qquad \delta_\epsilon \lambda = -\frac{1}{2\sqrt{2}} \left(\Gamma^M \partial_M \sigma + \tfrac{5}{4} M e^{\sigma} \right) \epsilon. \tag{4.2}$$

For configurations of the assumed form, and further assuming that ϵ depends only on y, the equations $\delta\psi = 0$ and $\delta\lambda = 0$ become

$$\begin{aligned} &0 = g^{-1}\epsilon' + \tfrac{1}{8} M e^{\sigma} \bar{\Gamma}_y \epsilon, \qquad 0 = \left(g^{-1} f' \bar{\Gamma}_y + \tfrac{1}{4} M f e^{\sigma} \right) \epsilon, \\ &0 = \left(g^{-1} \sigma' + \tfrac{5}{4} M e^{\sigma} \bar{\Gamma}_y \right) \epsilon, \end{aligned} \tag{4.3}$$

where the prime indicates differentiation with respect to y, and $\bar{\Gamma}$ are the *constant*, orthonormal frame basis, gamma matrices. To find non-zero solutions for ϵ we are now forced to suppose that ϵ has a definite 'chirality' in the sense that

$$\bar{\Gamma}_y \epsilon = \pm \epsilon. \tag{4.4}$$

We then find that

$$g^{-1} f' = \mp \tfrac{1}{4} M f e^{\sigma} \tag{4.5}$$

and that

$$g^{-1}\left(e^{-\sigma}\right)' = \pm\tfrac{5}{4}M. \tag{4.6}$$

Eliminating M from these equations we deduce that f is proportional to $e^{\sigma/5}$. As we are free to rescale the coordinates x^{μ}, we may choose the constant of proportionality to be unity, without loss of generality. Thus,

$$f = e^{\sigma/5}. \tag{4.7}$$

We are also free to choose $g(y)$ to be any function that is non-singular, where $f(y)$ is non-singular[4]. For example, the choice $g = f$ leads to a manifestly conformally flat form of the 8-brane metric. A solution in this form was given in Ref. [12]. We postpone a discussion of this solution until we have the general solution, to be given below. The choice of g that we shall make here is $g = f^{-1}$. In this case, use of the A_9 field equation $M' = 0$ in (4.6) yields

$$\partial_y^2\left(e^{-4\sigma/5}\right) = 0. \tag{4.8}$$

The general solution is given in terms of a harmonic function $H(y)$, the precise nature of which will be discussed shortly, i.e.

$$e^{-4\sigma/5} = H(y). \tag{4.9}$$

This leads to the 8-brane configuration

$$ds^2 = H^{-1/2}dx^{\mu}dx^{\nu}\eta_{\mu\nu} + H^{1/2}dy^2, \qquad e^{-4\sigma} = H^5, \qquad M = \pm H'. \tag{4.10}$$

where the prime indicates differentiation with respect to y. We have verified that this configuration is a solution of the full set of field equations. The Killing spinor ϵ is given by

$$\epsilon = H^{-1/8}(y)\epsilon_0, \qquad \Gamma_y\epsilon_0 = \pm\epsilon_0, \tag{4.11}$$

where ϵ_0 is a constant spinor.

It remains to determine the harmonic function H. Consider first the massive IIA theory for which the function M equals the (non-zero) constant m appearing in the lagrangian, which we may choose to be positive. In this case

$$H = \pm m(y - y_0) \tag{4.12}$$

for constant y_0, where the sign depends on the choice of 'chirality' of ϵ. However, H must be positive for real σ, so the spinor ϵ must change chirality at $y = y_0$. This is possible because ϵ blows up at $y = y_0$ [5]. Thus, the massive IIA theory has a solution for which

[4] i.e. where neither f nor f^{-1} vanish.

[5] Presumably, this is acceptable because the metric is also singular at $y = y_0$; in any case, we shall see below that this feature is not generic for the general 8-brane solution of the new IIA supergravity theory.

$$H = m|y - y_0|. \tag{4.13}$$

Note that this is a continuous function of y with a kink singularity at $y = y_0$, at which the curvature tensor has a delta function singularity.

In the new IIA theory we may suppose that M is only *locally* constant. The form of the function H in this case depends on the type of point singularity that we allow. The above example suggests that we should require H to be a continuous function of y. There are solutions for which H is discontinuous but they have δ'-type singularities of the curvature tensor, and we shall not consider them. In any case, the restriction to kink singularities produces physically sensible results, as we shall see. An example of a solution with a single kink singularity of H is

$$H = \begin{cases} -ay + b, & y < 0, \\ cy + b, & y > 0, \end{cases} \tag{4.14}$$

where a, b and c are non-negative constants. We adopt this as the basic single 8-brane solution. It can be interpreted as a domain wall separating regions with different values of M. The regions $y \to \pm\infty$ are at infinite affine distance. The solution therefore has two asymptotic regions relative to which an 8-brane charge, $Q_\pm$, may be defined as the value of M as $y \to \pm\infty$. For the above solution,

$$Q_+ = c, \qquad Q_- = a. \tag{4.15}$$

The constant b determines the value of σ, and hence the value of the string coupling constant e^σ at the 8-brane core. In particular, if $b = 0$, the string coupling constant goes to infinity at the core. Note that the solution (4.13) of the massive IIA theory is the special case for which $a = c$ and $b = 0$.

The multi 8-brane generalization of (4.14) with the same charges is found by allowing kink singularities of H at $n+1$ ordered points $y = y_0 < y_1 < y_2 < \ldots < y_n$. The function H is

$$H = \begin{cases} -a(y - y_0) + \sum_{i=1}^n \mu_i (y_i - y_0) + b, & y < y_0, \\ (c - \sum_{i=1}^n \mu_i)|y - y_0| + \sum_{i=1}^n \mu_i |y - y_i| + b, & y > y_0, \end{cases} \tag{4.16}$$

where μ_i are positive constants and a, b, c are non-negative constants.

The asymptotically left-flat or right-flat solutions are those for which $Q_- = 0$ or $Q_+ = 0$, respectively. The asymptotically flat solutions are those which are both asymptotically left-flat and right-flat. An example of an asymptotically flat three 8-brane solution is given by $H = \mu^2 ||y - y_0| - |y - y_1|| + \gamma^2$, where μ and γ are arbitrary constants.

If we now introduce a new variable $w(y)$, such that

$$\frac{dw}{dy} = H^{1/2}, \tag{4.17}$$

then the above 8-brane solution becomes

$$ds^2 = Z^{-1/3}(w)\left[dx \cdot dx + dw^2\right], \qquad e^{-\sigma} = Z^{5/6}(w), \tag{4.18}$$

where $Z(w)$ is a harmonic function of w, related to $H(y)$ by

$$Z(w) = H^{3/2}\big(y(w)\big). \tag{4.19}$$

The 8-brane solution of Ref. [12] is of this form. For example, the single 8-brane of that reference corresponds to the special choice of H in (4.14) with $b = 0$. This can be seen from the fact that the conformal factor of the solution of Ref. [12] blows up at the position of the 8-brane, while this is true of the above 8-brane solution only if $b = 0$.

If the above 8-brane solutions are to be considered as a field theory realization of the Dirichlet 8-brane of type-IIA superstring theory, then one would expect them to be related by T-duality to both 7-brane and 9-brane solutions of IIB supergravity. Consider first the Dirichlet 9-brane; its field theory realization is just $D = 10$ Minkowski space-time or, in the context of the S^1 compactified theory required for T-duality considerations, the product of S^1 with $D = 9$ Minkowski space-time. This space-time can also be regarded as an 8-brane solution. T-duality requires that the same solution results from direct (as against double) dimensional reduction of the 8-brane solution found above. This is indeed the case because compatibility with the KK reduction requires us to choose the harmonic function H to be constant, in which case $M = 0$. The dimensionally reduced solution is then precisely the product of S^1 with $D = 9$ Minkowski space-time.

Thus, the IIB 8-brane can be regarded as a T-dual of the IIB 9-brane. The more difficult task of determining the IIB 7-brane solutions to which the IIA 8-brane solutions are T-dual is what will occupy us for most of the remainder of this paper; it involves the construction of a new massive $D = 9$ supergravity, to which we now turn our attention.

5. Massive $D = 9$ $N = 2$ supergravity

The standard dimensional reduction to $D = 9$ of either the massless IIA supergravity theory or the IIB supergravity theory yields the massless $N = 2$ $D = 9$ supergravity theory [14] (see also Ref. [18]). Here we shall construct a massive $N = 2$ $D = 9$ supergravity theory. We shall do this in two ways. The first involves the massive IIA supergravity theory. At first sight it might seem that this theory cannot be dimensionally reduced to $D = 9$ because the product of $D = 9$ Minkowski space with S^1 is not a solution of the field equations. However, all we need is a solution with an abelian isometry and the massive IIA 8-brane is such a solution. This allows us to reduce the massive $D = 10$ IIA theory to $D = 9$ [6]. We shall then show that exactly the same theory can be found by a Scherk–Schwarz dimensional reduction of the IIB supergravity theory.

We begin by dimensionally reducing the massive IIA supergravity theory. Since we ultimately wish to make contact with the IIB theory via T-duality, it is convenient to

[6] An alternative, equivalent, procedure would be to make use of the new A_9 formulation of IIA supergravity to reduce to $D = 9$ in the standard way; the resulting $D = 9$ theory has an 8-form potential which can be traded for a cosmological constant.

use the conventions of Ref. [14], where the massless T-duality rules are given. Thus, the first step is to rewrite the results of Section 2 in the notation of Ref. [14]. The field content in $D = 10$ is given by

$$\{\hat{g}_{\hat{\mu}\hat{\nu}}, \hat{C}_{\hat{\mu}\hat{\nu}\hat{\rho}}, \hat{B}^{(1)}_{\hat{\mu}\hat{\nu}}, \hat{A}^{(1)}_{\hat{\mu}}, \hat{\phi}\}, \tag{5.1}$$

where the fields $\hat{C}$ and $\hat{A}^{(1)}$ are the RR sector fields. We refer to Ref. [14] for details of the notation, but we remark here that *in this section, only the metric signature is 'mostly minus'* and that the hats indicate $D = 10$ variables; the $D = 9$ variables resulting from the dimensional reduction will be without hats. Our starting point is the following (string-frame) action, obtained by translating (2.14) into the conventions of Ref. [14]:

$$I_{\mathrm{IIA}} = \tfrac{1}{2}\int_{\mathcal{M}_{10}} d^{10}x\,\sqrt{-\hat{g}}\Big\{e^{-2\hat{\phi}}\big[-\hat{R} + 4|d\hat{\phi}|^2 - \tfrac{3}{4}|\hat{H}^{(1)}|^2\big]$$
$$+\tfrac{1}{4}|\hat{F}^{(1)}_m|^2 + \tfrac{3}{4}|\hat{G}_m|^2 + \tfrac{1}{2}m^2\Big\} + \tfrac{1}{64}\int_{\mathcal{M}_{10}} \hat{G}^2_m\hat{H}^{(1)}. \tag{5.2}$$

Apart from the cosmological term, all m-dependent terms occur via the field-strength tensors of the RR fields. As explained in Section 2, the m-dependent terms within these curvature tensors are determined by the Stueckelberg-type symmetries, which now read

$$\delta\hat{B}^{(1)} = d\hat{\eta}^{(1)}, \qquad \delta\hat{A}^{(1)} = -\tfrac{1}{2}m\hat{\eta}^{(1)}, \qquad \delta\hat{C} = -m\hat{\eta}^{(1)}\hat{B}^{(1)}. \tag{5.3}$$

The m-dependence of the corresponding RR curvatures is given by

$$\hat{F}^{(1)}_m = \hat{F}^{(1)}_{m=0} + m\hat{B}^{(1)}, \qquad \hat{G}_m = \hat{G}_{m=0} + \tfrac{1}{2}m(\hat{B}^{(1)})^2. \tag{5.4}$$

The $m = 0$ part of the curvatures in the conventions now being used may be found in Ref. [14].

The field content of the massive $D = 9$ type-II theory is given by

$$\{g_{\mu\nu}, C_{\mu\nu\rho}, B^{(i)}_{\mu\nu}, A^{(i)}_\mu, \phi, k, \ell\}. \tag{5.5}$$

The RR sector fields are $C, B^{(2)}, A^{(1)}$ and ℓ. The action can be obtained by straightforward dimensional reduction of the ten-dimensional theory and is given by

$$I = \tfrac{1}{2}\int_{\mathcal{M}_9} d^9x\,\sqrt{g}\Big\{e^{-2\phi}\big[-R + 4|d\phi|^2 - \tfrac{3}{4}|H^{(1)}|^2$$
$$-|d\log k|^2 + \tfrac{1}{4}k^2|F^{(2)}|^2 + \tfrac{1}{4}k^{-2}|F(B)|^2\big] + \tfrac{1}{2}m^2k$$
$$-\tfrac{1}{2}k^{-1}|d\ell - mB|^2 + \tfrac{1}{4}k|F^{(1)}_m|^2 + \tfrac{3}{4}k|G_m|^2 - \tfrac{3}{4}k^{-1}|H^{(2)}_m|^2\Big\}$$
$$-\tfrac{1}{64}\int_{\mathcal{M}_{10}} G^2_mF(B) + 4G_mH^{(1)}H^{(2)}_m. \tag{5.6}$$

The m-dependent factors in the curvature tensors are determined by the following $D = 9$ Stueckelberg-type symmetries (which follow straightforwardly from the $D = 10$ rules):

$$\begin{aligned}
\delta B &= dA, \qquad \delta A^{(1)} = -\tfrac{1}{2}m\eta^{(1)} - mAA^{(2)}, \qquad \delta B^{(1)} = d\eta^{(1)} - A^{(2)}dA, \\
\delta B^{(2)} &= A^{(1)}dA + mAB^{(1)} + \tfrac{1}{2}m\eta^{(1)}B, \qquad \delta C = -m\eta^{(1)}\big(B^{(1)} + A^{(2)}B\big), \\
\delta\ell &= mA.
\end{aligned} \tag{5.7}$$

These Stueckelberg symmetries lead to the following (unique) modified curvatures for the RR fields:

$$\begin{aligned}
F^{(1)}_m &= F^{(1)}_{m=0} + \ell F^{(2)}_{m=0} + m\big(B^{(1)} - A^{(2)}B\big), \\
G_m &= G_{m=0} + \tfrac{1}{2}m(B^{(1)})^2 - mB^{(1)}A^{(2)}B, \\
H^{(2)}_m &= H^{(2)}_{m=0} - \ell H^{(1)}_{m=0} - mBB^{(1)}.
\end{aligned} \tag{5.8}$$

The expressions for the $m = 0$ curvatures may again be found in Ref. [14].

We now turn to the (massless) $D = 10$ type-IIB theory. Its field content is given by

$$\{\hat{\jmath}_{\hat\mu\hat\nu}, \hat{\mathcal{B}}^{(i)}_{\hat\mu\hat\nu}, \hat\ell, \hat\varphi, \hat{D}^{(+)}_{\hat\mu\hat\nu\hat\rho\hat\sigma}\}, \qquad i = 1,2. \tag{5.9}$$

The RR sector fields are $\hat{\mathcal{B}}^{(2)}, \hat{D}^{(+)}$ and $\hat\ell$. The action is given by[7]

$$\begin{aligned}
I_{\rm IIB} = \tfrac{1}{2}\int_{\mathcal{M}_{10}} d^{10}x\,\sqrt{-\hat{\jmath}}\Big\{& e^{-2\hat\varphi}\big[-\hat{R} + 4|d\hat\varphi|^2 - \tfrac{3}{4}|\hat{\mathcal{H}}^{(1)}|^2\big] \\
& -\tfrac{1}{2}|d\hat\ell|^2 - \tfrac{3}{4}|\hat{\mathcal{H}}^{(2)} - \hat\ell\hat{\mathcal{H}}^{(1)}|^2 - \tfrac{5}{6}|\hat{F}(D)|^2\Big\} \\
& -\tfrac{1}{96}\int_{\mathcal{M}_{11}} \epsilon^{ij}\hat{F}(D)\,\hat{\mathcal{H}}^{(i)}\hat{\mathcal{H}}^{(j)}.
\end{aligned} \tag{5.10}$$

The question now is whether, after dimensional reduction to $D = 9$, the massless $D = 10$ type-IIB theory can be mapped onto the massive $D = 9$ type-II theory found above. The standard reduction is given in Ref. [14] and leads to the massless theory in nine dimensions. Since one cannot add a cosmological constant to the $D = 10$ type-IIB theory, we have to change something in the standard reduction. Our guiding point will be the $D = 9$ Stueckelberg symmetries (5.7). Once we can reproduce these, the action, with the exception of the cosmological term, follows by symmetry.

We observe that from the IIA point of view the Stueckelberg Λ-transformation is just the $\underline{x}$-component of the $D = 10$ Stueckelberg symmetry. From the IIB point of view it should come from a general coordinate transformation in the $\underline{x}$-direction since we know that $\hat\xi^{\underline{x}} = \Lambda$ for $m = 0$ [14]. In order to reproduce the Stueckelberg Λ-transformations we should therefore introduce an extra $\underline{x}$-dependence in some of the fields of the $D = 10$ IIB theory. The only $D = 9$ fields that have an m-dependent Λ-transformation are ℓ and $B^{(2)}, A^{(1)}$. We find that these RR fields can be given the correct Λ-transformation provided we introduce the following additional dependence linear in $\underline{x}$:

[7] Strictly speaking, there is no action for the $D = 10$ type-IIB theory. However, when properly used, the given action leads to a well-defined action in $D = 9$. For more details about this point, see e.g. Ref. [19].

$$\hat{\ell} = \ell + m\underline{x}, \qquad \hat{\mathcal{B}}^{(2)}_{\mu\nu} = B^{(2)}_{\mu\nu} - B_{[\mu}A^{(1)}_{\nu]} + m\underline{x}\big(B^{(1)}_{\mu\nu} + B_{[\mu}A^{(2)}_{\nu]}\big),$$
$$\hat{\mathcal{B}}^{(2)}_{\underline{x}\mu} = -A^{(1)}_{\mu} + m\underline{x}A^{(2)}_{\mu}. \tag{5.11}$$

Note that the x-dependence in $\hat{\ell}$, which was introduced to reproduce the correct $D = 9$ Stueckelberg Λ-transformation, at the same time leads, via the kinetic term of $\hat{\ell}$, to the desired cosmological constant in $D = 9$! This establishes a relation between the cosmological constant and the Stueckelberg symmetries. Note also that, although the IIB RR fields $\hat{\ell}$ and $\hat{\mathcal{B}}^{(2)}$ depend on $\underline{x}$, all the $\underline{x}$-dependence drops out in the $D = 10$ IIB action. This can be seen by rewriting the ansatz for $\hat{\mathcal{B}}^{(2)}$ in the following equivalent form:

$$\hat{\mathcal{B}}^{(2)}_{\hat{\mu}\hat{\nu}} = \hat{\mathcal{B}}^{(2)}_{\hat{\mu}\hat{\nu},m=0} + m\underline{x}\hat{\mathcal{B}}^{(1)}_{\hat{\mu}\hat{\nu},m=0}. \tag{5.12}$$

Finally, we still have to reproduce the correct $\eta^{(1)}$ Stueckelberg symmetries. For $m = 0$ this symmetry is related to the following type-IIB gauge symmetry:

$$\delta\hat{\mathcal{B}}^{(i)} = d\hat{\Sigma}^{(i)}, \qquad \delta\hat{D} = \tfrac{3}{4}d\hat{\Sigma}^{(2)}\hat{\mathcal{B}}^{(1)} - \tfrac{3}{4}d\hat{\Sigma}^{(1)}\hat{\mathcal{B}}^{(2)}, \tag{5.13}$$

with $\hat{\Sigma}^{(i)} = \eta^{(i)}_{\mu}$. It turns out that the following $\underline{x}$-dependence in $\hat{\Sigma}^{(2)}$ reproduces the correct $\eta^{(1)}$ Stueckelberg symmetry given in (5.7):

$$\hat{\Sigma}^{(2)}_{\mu} = \eta^{(2)}_{\mu} + m\underline{x}\eta^{(1)}_{\mu}. \tag{5.14}$$

This equation also follows from the requirement that the ansatz for $\hat{\mathcal{B}}^{(2)}_{\mu\nu}$ be consistent with the $m = 0$ rule $\delta B^{(1)} = d\eta^{(1)}$.

We have therefore recovered by non-trivial dimensional reduction of IIB supergravity the massive $N = 2$ $D = 9$ supergravity found earlier from reduction of the massive IIA theory. It is of interest to see how this mechanism is related to the Scherk–Schwarz (SS) mechanism [15]. The essential ingredient in their method was a global $U(1)$ symmetry in the higher dimension. Let Q be the anti-hermitian generator of this $U(1)$ symmetry and let ∂ denote differentiation with respect to the KK coordinate. Then the SS mechanism can be summarized by the equation $\partial = mQ$. In our case the relevant $U(1)$ group acts on $\hat{\ell}$ (which is periodically identified) by a shift, so we should require $\partial\hat{\ell} = m$. The solution is $\hat{\ell} = \ell + m\underline{x}$, as above. The $U(1)$ transformation of the field strength 3-forms $\hat{\mathcal{H}}$ must be such that the action (5.6) is invariant, which determines the action of Q on these fields. Setting $\partial = mQ$ then yields a dependence of these field strengths on $\underline{x}$ that is consistent with the $\underline{x}$-dependence (5.12) of the 2-form potentials.

Thus, the dimensional reduction used above is essentially an application of the SS method. However, the implications are rather different in the present context. For example, in the reduction of $D = 4$ $N = 1$ supergravity to $D = 3$ using the global chiral $U(1)$ symmetry [15], the SS mechanism generates masses for the fermions but no scalar potential, thereby breaking supersymmetry. In contrast, in our case a cosmological constant is also generated and the full supersymmetry of the action is preserved by the reduction. The reason that supersymmetry is preserved can be traced to the fact that the fermions can be redefined in such a way that they are $U(1)$-invariant. The specific redefinition required for this is given in Appendix A for the Einstein-frame fields but since the dilaton is $U(1)$-invariant this result holds also for the string-frame fields. Alternatively, one can note that in the example given in Ref. [15] the chiral $U(1)$ acts on the supersymmetry parameter and this triggers the Higgs mechanism.

Having established that both the $D = 10$ massive type-IIA and massless type-IIB theory map onto the same $D = 9$ massive type-II supergravity theory, it is straightforward to determine the massive type-II T-duality rules. We consider here only the map from massive IIA to massless IIB. The $m = 0$ rules are given in Ref. [14]. We give here only the rules that receive an m-dependent correction. These are the following:

$$
\begin{aligned}
\hat{\ell} &= \hat{A}^{(1)}_{\underline{x}} + m\underline{x}, \\
\hat{\mathcal{B}}^{(2)}_{\mu\nu} &= \tfrac{3}{2}\hat{C}_{\mu\nu\underline{x}} - 2\hat{A}^{(1)}_{[\mu}\hat{B}^{(1)}_{\nu]\underline{x}} + 2\hat{g}_{\underline{x}[\mu}\hat{B}^{(1)}_{\nu]x}\hat{A}^{(1)}_{\underline{x}}/g_{\underline{xx}} + m\underline{x}\left(\hat{B}^{(1)}_{\mu\nu} + 2\hat{g}_{\underline{x}[\mu}\hat{B}^{(1)}_{\nu]\underline{x}}g_{\underline{xx}}\right), \\
\hat{\mathcal{B}}^{(2)}_{\underline{x}\mu} &= -\hat{A}^{(1)}_{\mu} + \hat{A}^{(1)}_{\underline{x}}\hat{g}_{\underline{x}\mu}/\hat{g}_{\underline{xx}} + m\underline{x}\hat{g}_{\underline{x}\mu}/\hat{g}_{\underline{xx}}.
\end{aligned}
\tag{5.15}
$$

6. The circularly symmetric IIB 7-brane

The massive T-duality rules derived in the previous section are expected to relate IIA 8-brane solutions to IIB 7-brane solutions. Compatibility of the latter with the KK ansatz implies that the 7-brane solution must have circular symmetry in the transverse directions. We therefore begin with a construction of the general IIB 7-brane solution of this type that also preserves half the supersymmetry.

The most general static circularly symmetric 7-brane metric is

$$
ds^2 = f^2(r)d\tilde{x}\cdot d\tilde{x} + a^2(r)\big(d\chi + \omega(r)dr\big)^2 + b^2(r)dr^2, \tag{6.1}
$$

where $\{\tilde{x}\}$ are the coordinates of 8-dimensional Minkowski space-time (the 7-brane worldvolume), the χ-coordinate is along the $U(1)$ Killing vector field and r is a radial coordinate. We are free to choose the function b, and we shall choose it such that $b = a$. Next, we change coordinate from χ to

$$
\underline{x} = \chi + \kappa(r), \tag{6.2}
$$

where the function κ is such that

$$
\omega = \frac{d\kappa}{dr}. \tag{6.3}
$$

Note that since χ was an angular coordinate, so also is $\underline{x}$. We may choose the identification such that

$$
\underline{x} \sim \underline{x} + 1. \tag{6.4}
$$

The metric now reads

$$ds^2 = f^2(r)\, d\tilde{x} \cdot d\tilde{x} + a^2(r)\left[d\underline{x}^2 + dr^2\right]. \tag{6.5}$$

To find supersymmetric solutions, we assume that

$$\hat{\ell} = \ell(r) + \tilde{m}\underline{x}, \qquad \hat{\varphi} = \hat{\varphi}(r) \tag{6.6}$$

where $\tilde{m}$ is piecewise constant, and we set the rest of the fields equal to zero.

Next, we substitute this ansatz into the (string frame) Killing spinor equations

$$\begin{aligned} \delta_\epsilon \psi &\equiv D\epsilon + \tfrac{1}{8} i e^{\hat{\varphi}}\left(\Gamma^M \partial_M \hat{\ell}\right)\Gamma\epsilon = 0, \\ \delta_\epsilon \lambda &= \tfrac{1}{4}\left(\Gamma^M \partial_M \hat{\varphi} + i e^{\hat{\varphi}} \Gamma^M \partial_M \hat{\ell}\right)\epsilon = 0, \end{aligned} \tag{6.7}$$

and assume that $\epsilon = \epsilon(r)$ where

$$\bar{\Gamma}_{\underline{x}}\epsilon(r) = \pm i \bar{\Gamma}_r \epsilon(r). \tag{6.8}$$

We thereby deduce that

$$\epsilon' \pm \tfrac{1}{8} e^{\hat{\varphi}} \tilde{m} \epsilon = 0 \tag{6.9}$$

and

$$\begin{aligned} &f' \pm \tfrac{1}{4} e^{\hat{\varphi}} f \tilde{m} = 0, \qquad \ell' = 0, \qquad a^{-1}a' \mp \tfrac{1}{4}\tilde{m} e^{\hat{\varphi}} = 0, \\ &\quad \hat{\varphi}' \pm e^{\hat{\varphi}} \tilde{m} = 0, \end{aligned} \tag{6.10}$$

where the prime indicates differentiation with respect to r.

Using the last two equations in (6.10), we have that

$$\partial_r^2\left(e^{-\hat{\varphi}}\right) = 0. \tag{6.11}$$

Thus we can set

$$e^{-\hat{\varphi}} = H(r), \tag{6.12}$$

where H is a harmonic function of r, of the type described in Section 4. The last of Eqs. (6.10) now yields

$$\tilde{m} = \pm H', \tag{6.13}$$

while the remainder of Eqs. (6.10) yields the full 7-brane solution in terms of H and three constants of integration, which can be removed by rescaling the coordinates and shifting $\hat{\ell}$. This solution is

$$\begin{aligned} &ds^2 = H^{-1/2}(r)\, d\tilde{x} \cdot d\tilde{x} + H^{1/2}(r)\left[d\underline{x}^2 + dr^2\right], \qquad e^{-\hat{\varphi}} = H(r), \\ &\quad \hat{\ell} = \pm H'(r)\underline{x}. \end{aligned} \tag{6.14}$$

The Killing spinor corresponding to this solution is given by

$$\epsilon = H^{-1/8}\epsilon_0, \qquad \bar{\Gamma}_{\underline{x}}\epsilon_0 = \pm i\bar{\Gamma}_r\epsilon_0. \tag{6.15}$$

We suggest that this 7-brane solution of IIB supergravity is the field theory realization of the Dirichlet 7-brane of type-IIB superstring theory. As a check on this interpretation we shall now verify that it is T-dual to the IIA 6-brane solution of Ref. [20]. To this end we take $\{\tilde{x}\} = (v^m, u)$, where v^m are coordinates for 7-dimensional Minkowski space-time (the 6-brane worldvolume), and also take the ignorable coordinate u to be an angular coordinate. We can then apply (massless) T-duality rules of Ref. [14], in the u-direction. This leads to the following solution of IIA supergravity:

$$\begin{aligned} ds^2 &= H^{-1/2}(r)\, dv\cdot dv + H^{1/2}(r)\left[du^2 + d\underline{x}^2 + dr^2\right], \qquad e^{-\hat{\phi}} = H^{3/4}(r), \\ \hat{A}^{(1)} &= \pm H'(r)\underline{x}du. \end{aligned} \tag{6.16}$$

This is precisely the IIA 6-brane solution in the form given in Ref. [21], except that in the general 6-brane solution the harmonic function H depends on all three 'transverse' variables $(u, \underline{x}, r)$. Thus, this is the form of the 6-brane compatible with a KK reduction to $D = 8$. This is an encouraging sign that the 7-brane will also be T-dual to a IIA 8-brane solution, since one expects the 6-brane and 8-brane to be equivalent on reduction to $D = 8$.

In order to show that this is indeed the case we need to establish the T-duality of the 7-brane to the 8-brane. We shall now show that the massive T-duality rules that we have given in Section 5 relate the IIA 8-brane of Section 4 to the IIB 7-brane given in (6.14). Although the general massive type-II T-duality rules are complicated, they become very simple for the special solutions considered here. Since

$$\hat{g}_{\underline{x}\mu} = \hat{C} = \hat{B}^{(1)} = \hat{A}^{(1)} = 0, \tag{6.17}$$

for our solutions, the massive type-II T-duality rules are

$$\hat{\jmath}_{\mu\nu} = \hat{g}_{\mu\nu}, \qquad \hat{\jmath}_{\underline{x}\underline{x}} = 1/\hat{g}_{\underline{x}\underline{x}}, \qquad \hat{\ell} = m\underline{x}, \qquad \hat{\varphi} = \hat{\phi} - \tfrac{1}{2}\log(-\hat{g}_{\underline{x}\underline{x}}). \tag{6.18}$$

To show that under the massive T-duality rules the IIA eight brane solution of Section 4 is T-dual to the IIB 7-brane solution, (6.14), we first make the change of notation $\{x^\mu\} = (\tilde{x}, \underline{x})$ and $y = r$. We then wrap the 8-brane in a compactifying direction, which we can choose to be $\underline{x}$. The 8-brane solution is then as follows:

$$\begin{aligned} ds^2 &= H^{-1/2}(r)d\tilde{x}\cdot d\tilde{x} + H^{-1/2}(r)d\underline{x}^2 + H^{1/2}(r)dr^2, \qquad e^{-4\sigma} = H^5(r), \\ M &= \pm H'(r), \end{aligned} \tag{6.19}$$

It is now straightforward to show that the T-duality rules, (6.18), applied to the $\underline{x}$-direction, take the IIA 8-brane solution to the IIB 7-brane solution. The equivalence of 8- and 7-branes is a non-trivial check of our massive T-duality rules.

7. Superstrings and supermembranes

The metric and dilaton (σ) of the RR p-brane solutions of $D = 10$ IIA or IIB supergravity were shown in Ref. [21], for $p \leq 6$, to be expressible in the form

$$ds^2 = H^{-1/2}d^2s_{p+1} + H^{1/2}dy \cdot dy, \qquad e^{4\sigma} = H^{3-p}, \tag{7.1}$$

where d^2s_{p-1} is the Minkowski $(p+1)$-metric, $dy \cdot dy$ is the Euclidean metric on the 'transverse' space $\mathbb{R}^{9-p}$, and H is a harmonic function on this space, apart from point singularities. Here we have found new IIB 7-brane solutions that are also of this form and we have shown that they are related by T-duality both to the IIA KK 6-brane and to the IIA 8-brane, of which we have also given the general solution preserving half the supersymmetry. This 8-brane solution can also be put in the above form. One advantage of this form of the solutions is that the T-duality between the RR p-branes and the RR $(p+1)$-branes, after compactification on S^1, is an almost immediate consequence of the T-duality rules of Ref. [14], at least for $p \leq 6$. The relationship between the 7-brane and the 8-brane solutions is more subtle, as we have seen, because it involves the comparison in $D = 9$ via a previously unknown *massive* $N = 2$ $D = 9$ supergravity theory.

So far, the context of our discussion has been that of supergravity rather than superstring theory. A new feature of the IIB superstring theory is its conjectured $Sl(2;\mathbb{Z})$ U-duality which requires, in particular, that the pseudoscalar $\hat{\ell}$ be periodically identified, i.e. that it takes values in S^1. Without loss of generality we can suppose that the identification is such that

$$\hat{\ell} \sim \hat{\ell} + 1. \tag{7.2}$$

Returning now to the ansatz (6.6), we note that since $\underline{x} \sim \underline{x} + 1$, the consistency of this ansatz requires $\tilde{m}$ to be an integer. Of course, since $\tilde{m}$ is not dimensionless, this result holds only for a particular choice of units. Such a choice is implicit in the choice of periodicity of $\underline{x}$. If the period is chosen to be R_B, which can be interpreted as the radius of the compact dimension, one finds that the unit of quantization of $\tilde{m}$ is $1/R_B$. That is[8],

$$\tilde{m} = \frac{n}{R_B} \tag{7.3}$$

for integer n. Recall now that the equivalence of the 7-brane with the 8-brane under T-duality requires that $m = \tilde{m}$. This means, assuming IIB U-duality, that the IIA 8-brane solution can be mapped to a IIB 7-brane solution by T-duality only if the cosmological constant m of the massive IIA theory is quantized as above, i.e. each time one passes

[8] Note added in proof: we have implicitly assumed that the IIB string coupling constant g_B is unity. As recently shown [28] the right hand side should be replaced by $n/g_B R_B$ when $g_B \neq 1$; the T-duality transformation between g_B and the IIA string coupling constant g_A then leads to a quantization condition of the form $m \sim n/(g_A\sqrt{\alpha'})$ in which the IIA mass parameter is expressed entirely in IIA terms.

through a IIA 8-brane the cosmological constant must jump by an integer multiple of basic unit $1/R_B$.

The single 8-brane solution should be related to the Dirichlet 8-brane of Ref. [2]. This is a string background in which open string states arise with fixed (Dirichlet) boundary conditions that are imposed in one space-like dimension at one or both ends of the string. These conditions restrict at least one of the end-points of open strings to lie in the nine-dimensional worldvolume of an 8-brane. The 8-brane couples to a 9-form gauge field with a 10-form field strength F_{10}. If the new IIA supergravity constructed here is indeed the effective field theory of the IIA superstring in the presence of this 10-form field strength, then it should be possible to recover the lagrangian (3.8) by string theory considerations. Neglecting terms of order B^2, which in any case follow from gauge invariance, the only term in (3.8) that is linear in F_{10} is proportional to

$$(\varepsilon F_{10})dA \cdot B. \tag{7.4}$$

This is the crucial term that has to be reproduced in string theory. There is a vertex operator in the RR sector of the type-IIA theory that couples a 10-form field strength to the worldsheet. This vertex operator has the form $F_{10}\bar{S}S$, where S is the space-time spinor worldsheet field of the space-time supersymmetric worldsheet action. There are non-trivial tree diagrams that mix F_{10} with fields from the RR and NSNS sectors, producing a term of the form (7.4), as required. The requirements of gauge invariance suggest that a more systematic consideration of string theory in the presence of D-branes would produce the full effective lagrangian (3.8).

Since all the p-brane solutions of $D = 10$ IIA supergravity for $p < 8$ can be viewed as arising from some 11-dimensional supermembrane theory, or 'M-theory' [22,21,23-25], it would be surprising if the 8-brane did not also have an 11-dimensional interpretation. The obvious possibility is that the $D = 10$ 8-brane is the double dimensional reduction of a $D = 11$ supersymmetric 9-brane. Such an object would be expected (see Ref. [1]) to carry a 9-form 'charge' appearing in the $D = 11$ supertranslation algebra as a central charge. This is possible because the 2-form charge normally associated with the $D = 11$ supermembrane is algebraically equivalent to a 9-form. It is not easy to see how to implement this idea, however, since there is no 'massive' $D = 11$ supergravity theory. One possibility is suggested by the recent interpretation [25] of the heterotic string as an $S^1/\mathbb{Z}_2$ compactified M-theory. Since the compactification breaks half the supersymmetry and the compactifying space is actually the closed interval, the two $D = 10$ space-time boundaries might be viewed as the worldvolumes of two $D = 11$ 9-branes.

Less ambitiously, one could try to relate the massive IIA supergravity theory to $D = 11$ supergravity via some lower dimension, in the same way that the IIB theory is related to it via reduction to $D = 9$. In fact, this can be done by compactification to $D = 8$. To see this we first observe that there is clearly a new massive $N = 2$ $D = 8$ supergravity theory obtainable *either* from the massive $N = 2$ $D = 9$ theory (by the same procedure used to obtain the latter from the massive IIA theory in $D = 10$) *or* from the massless $N = 2$ $D = 9$ theory by Scherk-Schwarz dimensional reduction (using the global $U(1)$ symmetry inherited from the $D = 10$ IIB theory). Thus, solutions of this massive $N = 2$

$D = 8$ supergravity theory should be liftable to $D = 9$ as solutions of *either* the massless *or* the massive $N = 2$ $D = 9$ supergravity theory. However, solutions of the latter are also solutions of the massive $D = 10$ IIA theory, while solutions of the former are also solutions of $D = 11$ supergravity.

In light of this we may now ask to what solution of $D = 11$ supergravity does the 8-brane solution of the massive IIA theory correspond? According to the above procedure we should first double dimensionally reduce the 8-brane to $D = 8$, where it can be interpreted as a 6-brane. This 6-brane solution can of course be lifted to $D = 9$ as a 7-brane solution of the massive $N = 2$ $D = 9$ supergravity theory, but we expect that it can also be lifted to $D = 9$ as a 6-brane solution of the *massless* $N = 2$ $D = 9$ theory which can then be lifted to $D = 10$ as the 6-brane solution of the massless IIA theory[9]. As shown in Ref. [21], this 6-brane solution is a non-singular solution of $D = 11$ supergravity analogous to the $D = 5$ KK monopole. Thus the M-theory interpretation of the massive IIA 8-brane would appear to be as the KK 6-brane, at least on compactification from $D = 11$ to $D = 8$.

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Appendix A. An $SL(2,\mathbb{R})$-formulation of $D = 10$ type-IIB supergravity

In Section 5 we used the supersymmetry transformations of type-IIB supergravity in ten dimensions in a rather simple form. In this appendix we present the relation of our formulation to that given in Ref. [26]. Since we make no reference to $D = 9$ fields in this section, we shall drop the hats on the $D = 10$ fields.

The scalars of IIB supergravity parametrize the $SU(1,1)/U(1)$ coset. They form an $SU(1,1)$ doublet, and under the local $U(1)$ they transform, with weight -1, as follows:

$$\phi'_\alpha = \exp(-i\Lambda)\phi_\alpha. \tag{A.1}$$

Here $\alpha = 1,2$ and ϕ_α satisfies $|\phi_1|^2 - |\phi_2|^2 = 1$. The complex fermions ψ_μ and λ have $U(1)$ weights $-1/2$ and $-3/2$ respectively.

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[9] It can also be considered as a 7-brane solution of the S^1-compactified IIB theory.

The first step, also worked out in Ref. [26], is to fix a $U(1)$ gauge by choosing ϕ_1 real:

$$\phi_1 = (\phi_1)^* = \frac{1}{\sqrt{1-\Phi^*\Phi}}, \qquad \phi_2 = \frac{\Phi}{\sqrt{1-\Phi^*\Phi}}. \tag{A.2}$$

This gauge choice is not invariant under $SU(1,1)$ or supersymmetry, which requires redefinition of these symmetries with compensating $U(1)$ transformations. The complex field Φ can be written as

$$\Phi(x) \equiv \frac{1+i\tau(x)}{1-i\tau(x)}, \qquad \tau(x) \equiv \ell(x) + ie^{-\varphi(x)}. \tag{A.3}$$

The $SL(2,\mathbb{R})$ transformations of τ are now given by

$$\tau' = \frac{c+d\tau}{a+b\tau}, \qquad ad-bc=1. \tag{A.4}$$

In Refs. [14,27] an action for the bosonic part of IIB supergravity was given in terms of the real scalars ℓ and φ, where φ was identified as the dilaton. Even though this bosonic action is simple, supersymmetry still is quite complicated if no further redefinitions are made. In particular, the variation of the gravitino still contains the composite $U(1)$-gauge field Q_μ, which is a complicated function of τ. Also, the transformation rule of λ is nonlinear.

The following redefinitions of the type-IIB fermions simplifies matters considerably:

$$\begin{aligned}\tilde\psi_\mu &= \exp(-\tfrac{1}{2}i\theta(x))\psi_\mu, \qquad \tilde\epsilon = \exp(-\tfrac{1}{2}i\theta(x))\epsilon,\\ \tilde\lambda &= \exp(-\tfrac{3}{2}i\theta(x))\lambda,\end{aligned} \tag{A.5}$$

where the function $\theta(x)$ is defined by

$$\exp(-2i\theta(x)) = \frac{1-i\tau}{1+i\tau^*}. \tag{A.6}$$

The $SL(2,\mathbb{R})$ transformations of $\tilde\psi, \tilde\epsilon, \tilde\lambda$ are as follows:

$$\begin{aligned}\tilde\lambda' &= \left(\frac{a+b\tau^*}{a+b\tau}\right)^{3/4}\tilde\lambda, \qquad \tilde\epsilon' = \left(\frac{a+b\tau^*}{a+b\tau}\right)^{1/4}\tilde\epsilon,\\ \tilde\psi' &= \left(\frac{a+b\tau^*}{a+b\tau}\right)^{1/4}\tilde\psi.\end{aligned} \tag{A.7}$$

Note that the redefined fermions are invariant under the abelian subgroup of $Sl(2;\mathbb{R})$, defined by setting $a=d=1$, $b=0$. This subgroup, which acts on τ as $\tau'=\tau+c$, is the group used for SS dimensional reduction in Section 5.

The redefinition (A.5) leads to the following simplified, Einstein frame, supersymmetry transformations (now omitting the tildes):

$$\delta e_\mu^a = \tfrac{1}{2}\bar\epsilon\Gamma^a\psi_\mu + \text{h.c.},$$

$$
\begin{aligned}
\delta\psi_\mu &= \mathcal{D}_\mu\epsilon + \tfrac{1}{4}i\epsilon e^{\varphi}\partial_\mu\ell - \tfrac{1}{192}i\Gamma^{(5)}\Gamma_\mu\epsilon F_{(5)} \\
&\quad - \tfrac{1}{16}\left(\Gamma_\mu\Gamma^{(3)} + 2\Gamma^{(3)}\Gamma_\mu\right)\epsilon^* e^{\varphi/2}\left(H^{(1)} - \ell H^{(2)} - ie^{-\varphi}H^{(2)}\right)_{(3)}, \\
\delta A_{\mu\nu\lambda\rho} &= i\bar{\epsilon}\Gamma_{[\mu\nu\lambda}\psi_{\rho]} + \text{h.c.} - 6\epsilon_{ij}B^{(i)}_{[\mu\nu}\delta B^{(j)}_{\lambda\rho]}, \\
\delta B^{(1)}_{\mu\nu} &= \tfrac{1}{2}\left(e^{-\varphi/2} + i\ell e^{\varphi/2}\right)\left(\bar{\epsilon}^*\Gamma_{[\mu}\psi_{\nu]} - \tfrac{1}{2}\bar{\epsilon}\Gamma_{\mu\nu}\lambda\right) + \text{h.c.}, \\
\delta B^{(2)}_{\mu\nu} &= \tfrac{1}{2}ie^{\varphi/2}\left(\bar{\epsilon}^*\Gamma_{[\mu}\psi_{\nu]} - \tfrac{1}{2}\bar{\epsilon}\Gamma_{\mu\nu}\lambda\right) + \text{h.c.}, \\
\delta\lambda &= \tfrac{1}{4}\Gamma^\mu\epsilon^*\left(\partial_\mu\varphi + ie^{\varphi}\partial_\mu\ell\right) + \tfrac{1}{8}\Gamma^{(3)}\epsilon e^{\varphi/2}\left(H^{(1)} - \ell H^{(2)} - ie^{-\varphi}H^{(2)}\right)_{(3)}, \\
\delta\ell &= ie^{-\varphi}\bar{\epsilon}\lambda^* + \text{h.c.}, \\
\delta\varphi &= \bar{\epsilon}\lambda^* + \text{h.c.}
\end{aligned}
\tag{A.8}
$$

The covariant derivative $\mathcal{D}\epsilon$ in the variation of the gravitino contains only the gauge field of local Lorentz transformations but no composite $U(1)$ gauge field Q_μ. Due to the redefinitions of the fermions, the only remnant of Q_μ is a single $e^{\varphi}\partial\ell$ term. The 3-forms $H^{(i)}$, $i = 1, 2$ are the field strengths of the 2-form gauge fields $B^{(i)}$. The field strength $F_{(5)}$ satisfies a self-duality condition, and is given by

$$
F_{\mu\nu\lambda\rho\sigma} \equiv \partial_{[\mu}A_{\nu\lambda\rho\sigma]} - 6\epsilon_{ij}B^{(i)}_{[\mu\nu}H^{(j)}_{\lambda\rho\sigma]}. \tag{A.9}
$$

The above transformation rules should still be completed with terms bilinear in the fermion fields. In principle these can be constructed from the results of Ref. [26], using the redefinitions given above.

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